### Discussant's Notes on Network-Oriented, Multi-regional, Walrasian, Fixed Point Models....

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#### **General Comments on the Paper**

- I do not have direct familiarity with the ArrowHead models so I cannot confirm the ease of use or other implementation issues. The ease of use, graphical approach and fast convergence needs to be demonstrated.
- The paper is generally well-written with clear simple examples.
- It would be useful to provide a clearer idea of how prices and quantities are determined in all of the nodes in the world gas and oil models as the network solution algorithm goes up and down the network in the iteration process.
- The use of object and visually-oriented code and user interface may be a significant advantage if verified.
- The 2 variable (and multi-variable) linear solution algorithm is identified , apparently, as unique. It is the well-known "Jacobi" algorithm and is only guaranteed to converge if the "A" matrix is diagonally dominant. It <u>may</u> converge in other cases, especially with relaxation introduced.

### Some Personal Historical Remarks

- GEMS was sold by DFI to the EIA in 1978 to develop the Long-term Energy Analysis Package (LEAP) which was intended to project the years following the final year projected by the Midterm Energy Forecasting System (MEFS)
  - LEAP was delivered incomplete and was never "the modeling system of choice" for EIA, to my knowledge
  - LEAP was used only in one outlook cycle before the program was eliminated at EIA
  - The ArrowHead system, as evolved, may be useful to EIA in the new proposed modeling – assertions need to be carefully reviewed and its features verified

### Arguments for Using ArrowHead -- 1

- Convergence is irrelevant it either it will or will not converge
  - Convergence is very relevant in my 40 years of energy modeling. The number of iterations, influenced by relaxation, are also relevant because the number of iterations increase, depending on how slowly the solution is allowed to change. How does the user select the " $\alpha$ "?
  - The selection of the stopping criteria is also critical if there are numerous solutions within the stopping criteria, potentially implying large differences in quantities for prices that are very close since only price seems to be used to determine convergence. You need both p and q convergence tests.
- Only ArrowHead network is parallelizable or parallelized.
  - Answer: GAMS is very much parallelizable, solvers like CPLEX and XPRESS can use multiple cores on the same machine in parallel for MIP solutions. See <u>http://www.gams.com/presentations/present\_gor08\_ws\_grid.pdf</u> also see: http://interfaces.gams-

software.com/doku.php?id=the\_gams\_grid\_computing\_facility

 NEMS is parallelized even though it is a mix of optimization, and linear and non-linear models

### Arguments for Using ArrowHead -- 2

- LP and Complementarity formulations require monolithic and unwieldy code to solve; problem: size may prohibit their solution
  - Size, when it relates to storage of non-zero coefficients, can become a size problem for any algorithm
  - With today's computing power and parallelization of both
    GAMS and the optimizers, this may be a non-issue
  - GAMS has made large strides becoming user friendly since the 1970's
  - Any model which is represented with sectoral and regional components (modularity of design) is probably parallelizable (e.g., NEMS run time has been cut by over 60%) using relatively simple partitions

### Arguments for Using ArrowHead -- 3

- Benefits of the network algorithm: no need for
  - Jacobian, Hessian in solution algorithm
    - You don't need these multi-dimensional derivatives in any LP or complementarity formulations; You may or may not need them in non-linear fixed point systems
  - No need for full rank , nxn, or for using Newton-Rhapson method of iteration to accelerate convergence
    - An LP matrix is virtually never square and never needs to be full column rank; similarly for complementarity formulations; the solvers take care of this automatically; no need for Newton's method in most problems
  - No need for contrived complementarity/optimization problem
    - There is nothing contrived about complementarity/optimization problems that EIA uses as far as I can see. Typically, the solution algorithm matches the way the modeled submarket works (e.g., electricity and refineries optimize).
  - The convergence algorithm is Robust
    - Convergence is no more robust than the Jacobi algorithm and not as robust as the Gauss-Seidel algorithm which tends to be faster on most applications
  - You don't need GAMS
    - Correct, you don't need GAMS. There are other matrix generation or problem generation languages, all intended to be user friendly.

### Arguments for Using GEMS -- 4

- LP and Complementarity formulations require monolithic and unwieldy code to solve; problem: size may prohibit their solution
  - There are well-known and efficient sparse matrix techniques which have become commonplace in the current state-of-the art optimizers and zeroes are not a problem. Nor are large numbers of identical coefficients which are stored and used efficiently.
- Network solution algorithm for network formulations is simple and clear, unlike the LP or complementarity formulation.
  - The network algorithm described seems simple and transparent. Solution algorithms for LP's and complementarity problems are more complex than large network problems. However, the LP solution user is never aware of that complexity and does not really need or benefit by knowing it.
  - An iterative solution algorithm of any size never provides the "exact solution" except by pure luck – the algorithm stops when the iterates are close enough (within stopping tolerance).

# **Clarifying Questions -- 1**

- Are the number of equations always equal to the number of unknowns in your network model – it appears that way from your write-up but that doesn't seem likely
- Generally, how are prices and quantities determined as the prices flow up and quantities flow down the network?
- How are market shares determined when you have multiple supply sources to satisfy a demand?
- Key parameters in the model? How are they developed? Are there market sharing formulations similar to the LEAP model? How are those parameters derived?
- How are energy demands/supplies represented? Demand/supply curves? Is there anything behind those? Level of detail? (Easy to change?) Investment in them? How are technologies and tech investment represented?

# **Clarifying Questions -- 2**

- The paper mentions a number of graphical interface improvements. How close to completion are they?
- How are regional regulations and laws or geopolitical constraints handled/represented?
- How is capacity investment decisionmaking represented?
- What distinguishes a Walrasian model from a partial or full equilbrium model?
- Why do you stress fixed point algorithms as if they were unique? They aren't unique, even in energy models and certainly not for numerical analysts.

## **Auxiliary Slides**

### Jacobi Iterations

### • The iterative scheme taken from Wikepedia

Given a square system of n linear equations:

$$A\mathbf{x} = \mathbf{b}$$

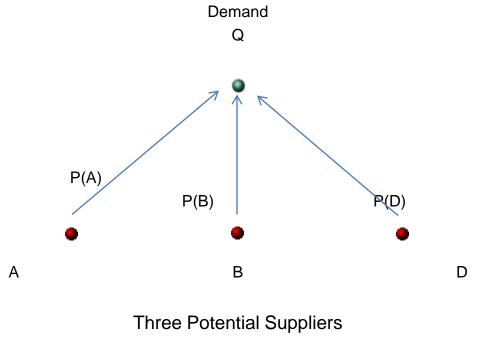
where:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

The element-based formula is thus:

$$x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j \neq i} a_{ij} x_j^{(k)} \right), \quad i = 1, 2, \dots, n.$$

#### How is demand satisfied and at what price



How much is taken from each supplier or what market share?

What is Q(A), Q(B), Q(D) or market shares Q(A)/Q etc